

Radial and angular wave function for hydrogen atom:-

In quantum mechanics, the wave function of the hydrogen atom are described by solutions to the Schrodinger equation in spherical coordinates. The wave function is characterized by the quantum numbers: principal quantum number n , azimuthal (or orbital angular momentum) quantum number l , and magnetic quantum number m .

The wave function can be separated into radial and angular parts :-

I. Radial wave function $R_{nl}(r)$

The radial part of the wave function depends on the distance r from the nucleus and is given by :-

$$R_{nl}(r) = A_{nl} \cdot \left(\frac{2Z}{na_0}\right)^{3/2} \left(\frac{r}{a_0}\right)^l \cdot e^{-Zr/na_0} \cdot \left(\frac{2Zr}{na_0}\right)^{2l+1} L_{n-l-1}^{2l+1}$$

where, Z is atomic number (1 for H-atom)

$a_0 =$ Bohr radius (approximately 0.529 \AA)

$L_{n-l-1}^{2l+1}(u)$ are the associated Laguerre polynomials.

$A_{nl} =$ normalization constant.

2. Angular wave function $Y_{lm}(\theta, \phi)$

The angular part is represented by spherical harmonics, given by:-

$$Y_{lm}(\theta, \phi) = N_{lm} \cdot P_l^m(\cos\theta) \cdot e^{im\phi}$$

where, N_{lm} is normalized constt.

$P_l^m =$ Associated Legendre polynomials

$\theta =$ polar angle, and ϕ is the azimuthal angle

Total wave function

The total wave function for the hydrogen atom can be expressed as the product of the radial and angular parts:-

$$\Psi_{nlm}(r, \theta, \phi) = R_{nl}(r) \cdot Y_{lm}(\theta, \phi)$$

eg:- The ground state wave function ($n=1, l=0, m=0$) of the hydrogen atom would have:-

→ Radial part:-

$$R_{10}(r) = \frac{1}{\sqrt{\pi a_0^3}} \cdot e^{-r/a_0}$$

→ Angular part:-

$$Y_{00}(\theta, \phi) = \frac{1}{\sqrt{4\pi}}$$

Thus, total wave function

$$\Psi_{100}(r, \theta, \phi) = \frac{1}{\sqrt{\pi a_0^3}} \cdot e^{-r/a_0} \cdot \frac{1}{\sqrt{4\pi}}$$

This gives complete description of H-atom wave function in the ground state.